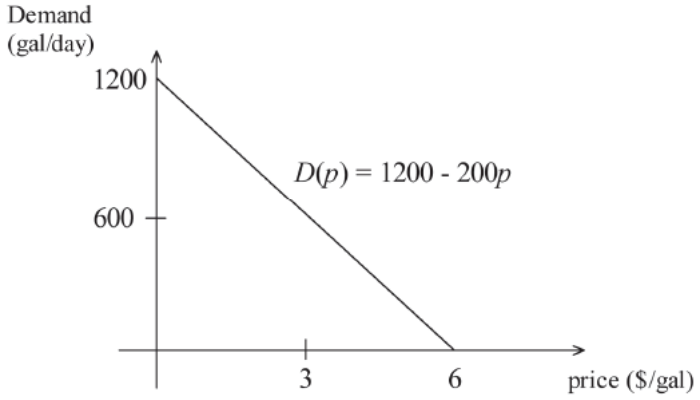
PLTL Calculus Session 6 – Elasticity and Economics

Economists use the term elasticity in relation to supply, demand, income, labor, capital, and other variables in systems with input and output. Basically, elasticity describes how changes in the input to a system are related to changes in the output. If that sounds familiar, it is because differentiation accomplishes the same task: measuring how a small change in the input variable affects change in the output variable.

Background Knowledge
Differentiation shortcut rules: constant, constant multiple, sum/difference, power, general exponentials, product, quotient

Linear demand
Normally, as the price of an item increases, the number of sales of that item generally decreases. This relationship is expressed in a demand function $D(p)$ that describes how demand depends on price. For this example, suppose a gas station has the demand function $D(p) = 1200 - 200p$; the graph is shown here.



1. According to this model, how many gallons of gas can the gas station owners expect to sell per month if the price is set at $4 per gallon? What about if the price is $3.50 per gallon? Which of these price points results in greater revenue for the gas station?

2. Evaluate $D'(p)$ and explain why your answer means that the demand function is decreasing. Why would you expect that demand functions should usually be decreasing functions?

3. For the comparison we made in Step 1, call the change in price $\Delta p = 4.00 - 3.50$; call the resulting change in the number of gallons sold ΔD, and write down this amount. Now, express your answers in terms of *percentages:* What is the *percent change* in price $\Delta p/p$, and what is the resulting *percent change* in the number of gallons sold $\Delta D/D$?

4. The elasticity in the demand, E, is the ratio of the percent change in demand to the percent change in price:

$$E = \frac{\Delta D/D}{\Delta p/p}$$

Compute the elasticity for the amounts of change in Step 3.

5. The elasticity is simplified by considering only very small changes in p and D. In this case, we can apply the definition of the derivative:

$$E = \lim_{\Delta p \to 0} \frac{\Delta D/D}{\Delta p/p} = \lim_{\Delta p \to 0} \frac{\Delta D}{\Delta p}\left(\frac{p}{D}\right) = \frac{dD}{dp}\left(\frac{p}{D}\right)$$

Prove that elasticity is a function of p only, and in this case find a formula for $E(p)$. Evaluate $E(3.5)$ and compare to your answer in Step 4.

6. The elasticity may be interpreted as the percent change in the demand that results from every one percent change in the price. For example if $E(p) = -2$, a one percent increase in price results in a two percent decrease in demand. For our gas example here, if the price of gasoline is \$4.50 and there is a 3.5% increase in the price, what is the elasticity and the corresponding percent change in the number of gallons of gas sold? Would this move be profitable for the gas station?

7. When $-\infty < E < -1$, the demand is said to be elastic, and when $-1 < E < 0$, the demand is said to be inelastic. Further, when $E = -\infty$ the demand is perfectly elastic, and when $E = 0$ the demand is perfectly inelastic. Essential goods such as basic food tend to have inelastic demand, but discretionary items such as electronics tend to have elastic demands. Using these examples, explain why you think we use the terms *elastic* and *inelastic*.

8. Graph the gasoline demand elasticity function for $0 \leq p < 6$. For what prices is the gasoline demand function elastic? For what prices is the gasoline demand function inelastic?

9. The demand for processed pork in Canada is described by the function $D(p) = 286 - 20p$. Graph this demand function, compute the elasticity, and graph the elasticity. For what prices is the demand function elastic? For what prices is the demand function inelastic?

10. Show that if a and b are positive real numbers, the general linear demand function $D(p) = a - bp$ has decreasing elasticity for $0 \leq p < a/b$. What is true about the elasticity when $p = a/b$?

Other Examples

1. Compute the elasticity for the exponential demand function $D(p) = ae^{-bp}$, where a and b are positive real numbers. For what values of p is the demand elastic? Inelastic?

2. Compute the elasticity for the demand function $D(p) = a/p^b$, where a and b are positive real numbers. For what values of p is the demand elastic? Inelastic?